



Bipolar Valued Intuitionistic Multi Fuzzy Normal Subnear-Ring of a Near-Ring

Dr. M. Mary Jansirani¹ & Dr. S. Vasantha Gowri²

¹Associate Professor, Department of Mathematics, SRM-Institute of Science and Technology (SRM Deemed to be University), Irungalur, Trichy-05, Tamil Nadu.

²Assistant Professor, Department of Mathematics, SRM-Institute of Science and Technology (SRM Deemed to be University), Irungalur, Trichy-05, Tamil Nadu.

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Abstract: In this paper, bipolar valued intuitionistic multi fuzzy normal subnear-ring of a near-ring is introduced and some theorems are stated and proved.

Keywords: Bipolar valued fuzzy subset, Bipolar valued multi fuzzy subset, Bipolar valued intuitionistic multi fuzzy subnear-ring, Bipolar valued intuitionistic multi fuzzy normal subnear-ring, Product and strongest bipolar valued intuitionistic multi fuzzy subnear-ring.

1. Introduction

In 1965, Zadeh [9] introduced the notion of a fuzzy subset of a universal set. Zhang [10, 11] introduced an extension of fuzzy sets named bipolar valued fuzzy sets in 1994 and bipolar valued fuzzy set was developed by Lee [3, 4]. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [4]. Anitha et al. [1, 2] introduced the bipolar valued fuzzy subgroup. Shyamala and Shanthi [7] have introduced the bipolar valued multi fuzzy subgroups of a group. Yasodara and Sathappan [8] defined the bipolar valued multi fuzzy subsemirings of a semi ring. Bipolar valued multi fuzzy subnear-ring of a near-ring has been introduced by Muthukumaran and Anandh [5]. In this paper, the concept of bipolar valued intuitionistic multi fuzzy normal subnear-ring of a near-ring is introduced and established some results.

2. Preliminaries

Definition 2.1 ([10]). A bipolar valued fuzzy set (BVFS) $\mathcal{B}$ in X is defined as an object of the form $\mathcal{B} = \{\langle x, \mathcal{B}^+(u), \mathcal{B}^-(u) \rangle / x \in X\}$, where $\mathcal{B}^+ : X \rightarrow [0,1]$ and $\mathcal{B}^- : X \rightarrow [-1,0]$. The positive membership degree $\mathcal{B}^+(u)$ denotes the satisfaction degree of an element x to the property

corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set $\mathcal{B}$.

Definition 2.2. A bipolar valued Intuitionistic fuzzy set (BVIFS) $\mathcal{B}$ in X is defined as an object of the form $\mathcal{B} = \{ \langle x, \mathcal{B}^+(\alpha(x), \beta(x)), \mathcal{B}^-(\alpha(x), \beta(x)) \rangle / x \in X \}$, where $\mathcal{B}^+ : X \rightarrow [0, 1]$ and $\mathcal{B}^- : X \rightarrow [-1, 0]$. The positive membership degree $\mathcal{B}^+(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set

Definition 2.3. A bipolar valued Intuitionistic multi fuzzy set (BVIMFS) $\mathcal{A}$ in X is defined as an object of the form

$$\mathcal{B} = \left\{ \langle x, \mathcal{B}_1^+(\alpha(x), \beta(x)), \mathcal{B}_2^+(\alpha(x), \beta(x)) \dots \dots \dots \mathcal{B}_n^+(\alpha(x), \beta(x)), \mathcal{B}_1^-(\alpha(x), \beta(x)), \mathcal{B}_2^-(\alpha(x), \beta(x)) \dots \dots \dots \mathcal{B}_n^-(\alpha(x), \beta(x)) \rangle / x \in X \right\},$$

where $\mathcal{B}_i^+ : X \rightarrow [0, 1]$ and $\mathcal{B}_i^- : X \rightarrow [-1, 0]$ for all i . The positive membership degree $\mathcal{B}_i^+(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}_i^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set $\mathcal{B}$.

Definition 2.4. Let $\mathcal{R}$ be a near ring. A BVIMFS $\mathcal{B}$ is said to be a bipolar valued intuitionistic multi-fuzzy subnear-ring $\mathcal{R}$ (BVIMFSNR) if the following conditions are satisfied, for all i

- i) $\mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x) - \alpha(y)), \mathcal{B}_i^+(\beta(x) - \beta(y))\}$
- ii) $\mathcal{B}_i^+(xy) \geq \min\{\mathcal{B}_i^+(\alpha(x), \beta(x)), \mathcal{B}_i^+(\alpha(y), \beta(y))\}$
- iii) $\mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x) - \alpha(y)), \mathcal{B}_i^-(\beta(x) - \beta(y))\}$
- iv) $\mathcal{B}_i^-(xy) \geq \min\{\mathcal{B}_i^-(\alpha(x), \beta(x)), \mathcal{B}_i^-(\alpha(y), \beta(y))\}$, for all x, y for all $x, y \in \mathcal{R}$

Definition 2.5. Let $\mathcal{R}$ be a near ring. A bipolar valued intuitionistic multi fuzzy subnear-ring of $\mathcal{R}$ is said to be a bipolar valued intuitionistic multi fuzzy normal subnear-ring (BVIMFNSNR) of $\mathcal{R}$ if

- i) $\mathcal{A}_i^+(\alpha(x + y), \beta(x + y)) = \mathcal{A}_i^+(\alpha(y + x), \beta(y + x))$
- ii) $\mathcal{A}_i^-(\alpha(x + y), \beta(x + y)) = \mathcal{A}_i^-(\alpha(y + x), \beta(y + x))$
- iii) $\mathcal{A}_i^+(\alpha(xy), \beta(xy)) = \mathcal{A}_i^+(\alpha(yx), \beta(yx))$
- iv) $\mathcal{A}_i^-(\alpha(xy), \beta(xy)) = \mathcal{A}_i^-(\alpha(yx), \beta(yx))$ for all x, y in $\mathcal{R}$ and for all i .

Definition 2.6. Let $\mathcal{A} = (\mathcal{A}_1^+ \mathcal{A}_2^+ \dots \dots \dots \mathcal{A}_n^+, \mathcal{A}_1^- \mathcal{A}_2^- \dots \dots \dots \mathcal{A}_n^-)$ be a bipolar valued intuitionistic multi fuzzy subset in a set $\mathcal{S}$, the strongest bipolar valued intuitionistic fuzzy relation on $\mathcal{S}$ that is bipolar valued intuitionistic multi fuzzy relation on $\mathcal{A}$ is

$$\mathcal{V} = \left\{ \langle \alpha(xy), \beta(xy), \mathcal{V}_1^+(\alpha(xy), \beta(xy)) \dots \dots \dots \mathcal{V}_n^+(\alpha(xy), \beta(xy)), \mathcal{V}_1^-(\alpha(xy), \beta(xy)) \dots \dots \dots \mathcal{V}_n^-(\alpha(xy), \beta(xy)) \rangle / x, y \in \mathcal{S} \right\}$$

Where $\mathcal{V}_i^+(\alpha(xy), \beta(xy)) = \min\{\mathcal{A}_i^+(\alpha(xy), \beta(xy))\}$ and

$$\mathcal{V}_i^-(\alpha(xy), \beta(xy)) = \max\{\mathcal{A}_i^-(\alpha(xy), \beta(xy))\} \quad \text{for all } x, y \in \mathcal{S} \text{ and for all } i$$

3. Properties

Definition 3.1. Let $\mathcal{R}$ be a near ring and $\mathcal{B}$ be an bipolar valued Intuitionistic multi fuzzy subset of a ring $\mathcal{R}$. Then $\mathcal{B}$ is called an bipolar valued intuitionistic multi fuzzy right ideal of $\mathcal{R}$ if $\mathcal{B}$ is an bipolar valued intuitionistic multi fuzzy sub near ring of $\mathcal{R}$ and satisfies for all $x, y \in \mathcal{R}, i = 1, 2, \dots$

- i) $\mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\}$
 $\mathcal{B}_i^+(x - y)I \leq \max\{I\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\}$
- ii) $\mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\}$
 $\mathcal{B}_i^-(x - y)I \geq \min\{I\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\}$
- iii) $\mathcal{B}_i^+((x + i)y - xy) \geq \mathcal{B}_i^+(i) \quad \& \quad \mathcal{B}_i^+((x + i)y - xy)I \leq \mathcal{B}_i^+(i)I$
 $\mathcal{B}_i^-((x + i)y - xy) \leq \mathcal{B}_i^-(i) \quad \& \quad \mathcal{B}_i^-((x + i)y - xy)I \geq \mathcal{B}_i^-(i)I$
- iv) $\mathcal{B}_i^+(y + x - y) \geq \mathcal{B}_i^+(\alpha(x), \beta(x)) \quad \& \quad \mathcal{B}_i^+(y + x - y)I \leq \mathcal{B}_i^+(\alpha(y), \beta(y))I$
 $\mathcal{B}_i^-(y + x - y) \leq \mathcal{B}_i^-(\alpha(x), \beta(x)) \quad \& \quad \mathcal{B}_i^-(y + x - y)I \geq \mathcal{B}_i^-(\alpha(y), \beta(y))I$

Definition 3.2. Let $\mathcal{R}$ be a near ring and $\mathcal{B}$ be an bipolar valued Intuitionistic multi fuzzy subset of a ring $\mathcal{R}$. Then $\mathcal{B}$ is called an bipolar valued intuitionistic multi fuzzy left ideal of $\mathcal{R}$ if $\mathcal{B}$ is an bipolar valued intuitionistic multi fuzzy sub near ring of $\mathcal{R}$ and satisfies for all $x, y \in \mathcal{R}, i = 1, 2, \dots$

- i) $\mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\}$
 $I\mathcal{B}_i^+(x - y) \leq \max\{I\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\}$
- ii) $\mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\}$
 $I\mathcal{B}_i^-(x - y) \geq \min\{I\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\}$
- iii) $\mathcal{B}_i^+(\alpha(xy), \beta(xy)) \geq \mathcal{B}_i^+(\beta(xy)) \quad \& \quad I\mathcal{B}_i^+(\alpha(xy)) \leq I\mathcal{B}_i^+(\alpha(xy))$
 $\mathcal{B}_i^-(\alpha(xy)) \leq \mathcal{B}_i^-(\beta(xy)) \quad \& \quad I\mathcal{B}_i^-(\alpha(xy)) \geq I\mathcal{B}_i^-(\alpha(xy))$
- iv) $\mathcal{B}_i^+(y + x - y) \geq \mathcal{B}_i^+(\alpha(x), \beta(x)) \quad \& \quad I\mathcal{B}_i^+(y + x - y) \leq I\mathcal{B}_i^+(\alpha(y), \beta(y))$
 $\mathcal{B}_i^-(y + x - y) \leq \mathcal{B}_i^-(\alpha(x), \beta(x)) \quad \& \quad I\mathcal{B}_i^-(y + x - y) \geq I\mathcal{B}_i^-(\alpha(y), \beta(y))$

Proposition 3.3. If an bipolar valued intuitionistic multi fuzzy subsets of $\mathcal{R}$ satisfies the properties

$$\mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\}, \mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\}$$

then

- i) $\mathcal{A}_i^+(0) \geq \mathcal{A}_i^+(\alpha(x)) \quad \& \quad \mathcal{A}_i^-(0) \leq \mathcal{A}_i^-(\alpha(x))$
- ii) $\mathcal{A}_i^+(-\alpha(x)) = \mathcal{A}_i^+(\alpha(x)) \quad \& \quad \mathcal{A}_i^-(-\alpha(x)) = \mathcal{A}_i^-(\alpha(x))$
 $x, y \in \mathcal{R} \quad \& \quad i = 1, 2, \dots$

Proof

We have that for any $x \in \mathcal{R}$, $i=1,2,\dots$

$$\begin{aligned} \text{i) } \mathcal{A}_i^+(0) &= \mathcal{A}_i^+(\alpha(x) - \alpha(x)) \\ &\geq \min\{\mathcal{A}_i^+(\alpha(x)), \mathcal{A}_i^+(\alpha(x))\} \\ &= \mathcal{A}_i^+(\alpha(x)) \end{aligned}$$

$$\text{Hence } \mathcal{A}_i^+(0) \geq \mathcal{A}_i^+(\alpha(x))$$

$$\begin{aligned} \mathcal{A}_i^-(0) &= \mathcal{A}_i^-(\alpha(x) - \alpha(x)) \\ &\leq \max\{\mathcal{A}_i^-(\alpha(x)), \mathcal{A}_i^-(\alpha(x))\} \\ &= \mathcal{A}_i^-(\alpha(x)) \end{aligned}$$

$$\text{Hence } \mathcal{A}_i^-(0) \leq \mathcal{A}_i^-(\alpha(x))$$

$$\begin{aligned} \text{ii) } \mathcal{A}_i^+(-\alpha(x)) &= \mathcal{A}_i^+(0 - \alpha(x)) \\ &\geq \min\{\mathcal{A}_i^+(0), \mathcal{A}_i^+(\alpha(x))\} \\ &= \mathcal{A}_i^+(\alpha(x)) \end{aligned}$$

$$\text{Hence } \mathcal{A}_i^+(-\alpha(x)) = \mathcal{A}_i^+(\alpha(x))$$

$$\begin{aligned} \mathcal{A}_i^-(-\alpha(x)) &= \mathcal{A}_i^-(0 - \alpha(x)) \\ &\leq \max\{\mathcal{A}_i^-(0), \mathcal{A}_i^-(\alpha(x))\} \\ &= \mathcal{A}_i^-(\alpha(x)) \end{aligned}$$

$$\text{Hence } \mathcal{A}_i^-(-\alpha(x)) = \mathcal{A}_i^-(\alpha(x))$$

Proposition 3.4.

If $\mathcal{B} = (\mathcal{B}_1^+, \mathcal{B}_2^+ \dots \dots \mathcal{B}_n^+, \mathcal{B}_1^-, \dots \dots \mathcal{B}_n^-)$ and $\mathcal{C} = (\mathcal{C}_1^+, \mathcal{C}_2^+ \dots \dots \mathcal{C}_n^+, \mathcal{C}_1^-, \mathcal{C}_2^- \dots \dots \mathcal{C}_n^-)$ are two BVIMFNSNR with degree n of a near ring $\mathcal{R}$, then their intersection $\mathcal{B} \cap \mathcal{C}$ is a BVIMFNSNR of N.

Proof

Let $\mathcal{D} = \mathcal{B} \cap \mathcal{C}$, by two BVIMFNSRs with degree n of a near ring then their intersection $\mathcal{B} \cap \mathcal{C}$ is a BVIMFNSNR of N. Let $\mathcal{D}$ is a BVIMFNSNR of the near ring N.

Let $u, v \in N$, for all i

$$\begin{aligned} \mathcal{D}_i^+(u + v) &= \min\{\mathcal{B}_i^+(u(x) + v(x)), u(y) + v(y)\}, \mathcal{C}_i^+(u(x) + v(x)), u(y) + v(y)\} \\ &= \mathcal{D}_i^+(u + v) \text{ for every } u, v \in N \end{aligned}$$

$$\begin{aligned} \mathcal{D}_i^+(uv) &= \min\{\mathcal{B}_i^+(u(xy), v(xy)), \mathcal{C}_i^+(u(xy), v(xy))\} \\ &= \mathcal{D}_i^+(uv) \text{ for every } u, v \in N \end{aligned}$$

$$\begin{aligned} \mathcal{D}_i^-(u + v) &= \min\{\mathcal{B}_i^-(u(x) + v(x)), u(y) + v(y)\}, \mathcal{C}_i^-(u(x) + v(x)), u(y) + v(y)\} \\ &= \mathcal{D}_i^-(u + v) \text{ for every } u, v \in N \end{aligned}$$

$$\begin{aligned} \mathcal{D}_i^-(uv) &= \min\{\mathcal{B}_i^-(u(xy), v(xy)), \mathcal{C}_i^-(u(xy), v(xy))\} \\ &= \mathcal{D}_i^-(uv) \text{ for every } u, v \in N \end{aligned}$$

$\mathcal{B} \cap \mathcal{C}$ is a BVIMFNSNR of the near ring N .

4. Conclusion

Bipolar valued multi fuzzy normal subnear-ring of a near-ring is defined and their properties are proved. We can extend these concepts into many algebraic systems.

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