



De Morgan's Theorem And The Dynamism Of Negation In Human Systems

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Abstract

This paper advances a grounded interpretation of De Morgan's laws by examining their operative significance beyond formal logic and into the domains of language, cognition, and social systems. Building on the foundational work of Augustus De Morgan, it argues that the classical equivalences governing the transformation of conjunction and disjunction under negation are not merely static algebraic identities, but dynamic structures whose application within human systems reveals important limitations and productive tensions. By analytic method of research this paper seeks to demonstrate that the behavior of negation in natural language and human reasoning departs significantly from its idealized logical form. Phenomena such as negation-raising, coordination asymmetries, and context-sensitive interpretation expose a structural "fracture" between formal duality and lived meaning. The paper also situates these tensions within broader philosophical concerns about the limits of formal systems, drawing on insights associated with Kurt Gödel to argue that the application of De Morgan's framework to human and institutional contexts inevitably encounters incompleteness. In contemporary settings—ranging from legal reasoning to algorithmic governance—negation emerges not as a purely formal operation but as a socially embedded act shaped by interpretation, intention, and ethical judgment. Ultimately, the study contends that De Morgan's laws should be understood as foundational yet incomplete schemata that require supplementation by contextual, cognitive, and normative dimensions.

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1. Introduction

In the world of logic, mid-nineteenth century was dominated by a growing interest in formalizing and systematizing human rationality beyond what its adherents saw as the limits of Aristotelian syllogistic logic. Meanwhile, during the latter half of this industrial revolution, mathematicians like George Boole and Augustus De Morgan began to inject their own logic with the same precision and algebraic formality that characterized mathematics. The work of De Morgan, a polymath whose interests ranged across mathematics, history and logic, was instrumental in this development. His most lasting contribution, expressed in what we now call De

Morgan's Laws, supplied a framework for understanding the interplay of negation with the basic logical connectives and: conjunctive ('and') or: disjunctive ('or'). These two equations seem at first to be beautiful, symmetrical identities that hold directly and universally in all reasoning contexts. They express a basic duality, stating that the negation of a union is equal to the intersection of the negations, and vice versa. This rigorous beauty has etched De Morgan's Theorem into the rules of some of the most important games in computing and electrical engineering, in which exactitude is often key.

But, the superficial simplicity that such laws represent masks a complexity of quite different order when they encounter 'mess', richly-contexted human systems. The experiential dynamics of negation: how it can do its work, and spread, and transform indefinitely what is meant to natural languages as well as in cognitive processing or communicative acts imply that these formal identities encounter high resistance—and thus limitations. Just as Kurt Gödel proved how any powerful enough formal system will contain some true statements it cannot prove according to its own axioms, the use of De Morgan's logic on human communication and social intention often discloses a "Gödelian" crack. This break demonstrates that the negation of a whole' is seldom apt to be a mere arithmetical summation of its negated parts' where there are human understanding, context and agency. This essay starts to consider this 'Gödelian' dimension of De Morgan's project, suggesting at the level of rhetoric at least, that it is never possible to fully contain the intricacies of human negation within a closed system defined only by rules. We argue that the dynamism of negation is not a horrid characteristic of thought in need of remedying, but a necessary and celebrated aspect of cognitive life, without which one could not negotiate or determine the porous perimeter for truth, justice, and sense that exists within our constantly changing world.

2. De Morgan's Formal Framework and Its Historical Context

Augustus De Morgan's intellectual project was deeply rooted in a vision of logic as a "mental dynamics," a study not merely of static truths but of the active operations of the mind in reasoning. In his groundbreaking 1847 treatise, *Formal Logic: or, the Calculus of Inference, Necessary and Probable*, De Morgan laid the groundwork for an algebraic approach to logic that would profoundly influence subsequent developments in the field.¹ His work sought to liberate logic from the strictures of Aristotelian categories, expanding its scope to encompass a broader range of inferences and relationships. Central to this endeavor were the laws that bear his name, which provide the formal rules for the interconversion of conjunction and disjunction through negation. These laws are typically articulated in two canonical forms:

Law	Formal Expression	Natural Language Equivalent
Negation of Conjunction	$\neg(A \wedge B) \Leftrightarrow (\neg A \vee \neg B)$ This is also represented as ff: $\neg(A \wedge B) \iff (\neg A \vee \neg B)$	"It is not the case that both A and B are true" is logically equivalent to "Either A is false, or B is false, or both are false."
Negation of Disjunction	$\neg(A \vee B) \Leftrightarrow (\neg A \wedge \neg B)$ This is also represented as ff: $\neg(A \vee B) \iff (\neg A \wedge \neg B)$	"It is not the case that either A or B is true" is logically equivalent to "Both A is false and B is false."

These identities establish a fundamental "duality" within logic, illustrating how conjunction and disjunction are intrinsically linked and can be transformed into one another via the operation of negation. This concept of duality was not entirely novel; its roots can be traced back to medieval scholastic logic, particularly in the work of William of Ockham, who recognized similar equivalences. However, De Morgan provided the first rigorous algebraic formulation, integrating these principles into a broader calculus of logic. As Daniel Devidi, a

¹ Augustus De Morgan, *Formal Logic: or, the Calculus of Inference, Necessary and Probable* (London: Taylor and Walton, 1847), 7.

philosopher of logic, observes, a negation that consistently satisfies these four De Morgan laws is often termed "Ockham negation," underscoring its role in simplifying and structuring logical expressions.²

For De Morgan, the significance of these laws extended beyond mere algebraic manipulation. They were instrumental in defining the "Universe of Discourse", a concept he introduced to delineate the specific domain within which a given set of names or propositions holds meaning.³ Negation, in this context, becomes the act of identifying what lies outside this defined universe, and De Morgan's laws provide the precise mechanism for mapping this transition. This formalization offered a powerful tool for clarifying logical arguments and for understanding the boundaries of conceptual categories.

The contradiction between the formal rigour of his laws and the subtleties of human language was, however, not lost on De Morgan. He did, however, note that these formal rules were "not properly adapted to our mode of using language", which is perhaps the earliest warning regarding the difficulties one faces when transferring logical formalism over to the fluid and context-dependent sphere of human communication.⁴ De Morgan himself speculated on the nature of a "mental dynamics" which would account for why some logical operations seemed to be more intuitive or natural than others and which implied a cognitive dimension to logical thought that went beyond formalism *per se*. This tension between the "architecture of certainty" represented by De Morgan's formal laws and the "dynamism of application" in human thought and language creates conditions for structural rifts manifesting themselves in linguistics and cognitive science as we discuss further on.

3. The Linguistic Fracture: Negation and the Dynamism of Coordination in Natural Language

The transition from the pristine, axiomatic world of formal logic to the rich, often ambiguous, domain of natural language reveals a significant divergence in the behavior of logical operators. While formal logic posits a straightforward isomorphism between its connectives ($A \wedge B$, $A \vee B$) and their natural language counterparts ("and," "or"), the reality of human communication is far more complex. In natural language, "and" and "or" are not merely truth-functional operators; they are imbued with pragmatic, temporal, causal, and social nuances that profoundly influence their interpretation. This complexity is particularly evident in the "dynamism" of negation—the intricate ways in which the word "not" interacts with coordinated structures, often challenging the direct application of De Morgan's laws.

Linguistic research has extensively explored phenomena such as "NEG-raising" and the distribution of negation over coordinated phrases, demonstrating that natural language frequently deviates from the straightforward equivalences predicted by De Morgan's Theorem. Diego Gabriel Krivochen, in his work on "De Morgan's laws and NEG-raising," argues that the applicability of these laws to natural language structures is constrained by independently motivated operations of grammar and the inherent properties of linguistic coordination.⁵ Krivochen highlights a crucial distinction between different types of coordination, which he terms *et-coordination* and *que-coordination*, to illustrate this point. *Et-coordination* refers to structures where the conjuncts are perceived as distinct, accessible entities, allowing negation to distribute more readily across them. For example, in the sentence "Mary doesn't write novels and compose lieder," the negation can be understood as applying to each conjunct individually, aligning with De Morgan's law for the negation of a conjunction: $\neg(N \wedge L) \Leftrightarrow (\neg M \vee \neg L)$. This can be represented as following: $\neg(M \wedge L) \rightarrow (\neg M \vee \neg L)$. This suggests that Mary either doesn't write novels, or doesn't compose lieder, or both. Note that M represents Mary while L represents Lieder.

Conversely, *que-coordination* describes instances where the conjoined phrase functions as a single, atomic semantic object, resisting the decomposition of negation to its individual parts. Consider the example, "The sudden rise and fall of the market has economists worried." Here, the negation would typically apply to the entire event of "the sudden rise and fall," rather than negating "the sudden rise" and "the fall" independently. In such cases, the negation of the whole does not necessarily distribute to the parts because the parts do not maintain independent semantic existence within the "Universe of Discourse" established by the coordinated phrase. This linguistic phenomenon underscores a critical "Gödelian limit" in the formal modeling of natural language: a purely formal, truth-conditional model of communication often remains blind to the rich subtext,

² Daniel Devidi, "Negation: Philosophical Aspects," in *Concise Encyclopedia of Philosophy of Language and Linguistics*, ed. Alex Barber and Robert J. Stainton (Elsevier, 2006), 533.

³ De Morgan, *Formal Logic*, 41.

⁴ Augustus De Morgan, *On the Syllogism: and Other Logical Writings*, ed. Peter Heath (London: Routledge, 2015), 106.

⁵ Diego Gabriel Krivochen, "De Morgan's Laws and NEG-Raising: A Syntactic View," *Linguistic Frontiers* 9, no. 1 (2018): 1-12.

implicature, and contextual nuances that are central to human understanding. As John Inah Ukam observes in his analysis of formalism in communication, human interaction is an "open system" characterized by contextual sensitivity and emergent creativity, which formal models struggle to capture.⁶

Furthermore, the interaction between negation and coordination in natural language is often influenced by pragmatic factors, such as speaker intent and shared knowledge. The choice between expressing a negation as a negated conjunction or a disjunction of negations can convey subtle differences in emphasis or scope. For instance, while "It is not true that John is rich and famous" is logically equivalent to "John is not rich or John is not famous," the former might imply a denial of the combined state, while the latter might highlight alternative possibilities. This pragmatic dimension adds another layer of complexity to the dynamism of negation, moving beyond mere syntactic distribution. In legal contexts, the precise interpretation of a negated conjunction or disjunction can have profound implications. A legal text stating "The contract is void if conditions A and B are not met" $\neg(A \wedge B)$ might be interpreted differently than "The contract is void if condition A is not met or condition B is not met" $(\neg A \vee \neg B)$, even though they are formally equivalent. The legal "negation" of a contract often hinges on whether the intent behind the coordinated conditions was collective or distributive—a distinction that De Morgan's algebraic identities alone cannot fully resolve, requiring human interpretation and judicial discretion. This highlights how the formal architecture of duality, while powerful, requires contextual grounding to be fully operationalized in the fluid landscape of human language and law.

4. The Cognitive Burden: The Mental Dynamics of Logical Exchange

The dynamism which is part of De Morgan's Theorem is not just an abstract spitbubble figment that has to be constructed in linguistic analysis, but a salient reality in cognitive science, where the operation of logical relations in the mind by the human mind itself is considered. An extensive amount of empirical evidence points to a striking "asymmetry" between the effort to process negations applied to conjunctions and disjunctions. If De Morgan's laws were nothing but static, algebraic fodder, then we would intuitively expect that the mental effort required to comprehend $\neg(A \wedge B)$ should be roughly equivalent to the mental effort exerted in trying to understand $\neg(A \vee B)$. But this anticipation tends to run counter to the experimental evidence.

Macbeth et al.'s human studies, for example, show that the negation of a conjunction is dramatically harder to process and accept/identify than the negation of a disjunction.⁷ This "cognitive burden" observed, in turn, means that the kind of "exchange" mechanism proposed by De Morgan's laws (an "and" becoming an "or" (or conversely) [and negation distributing]) is not a lightning-fast operation within the human mind. Instead, this is likely a cognitive process with friction of substantial magnitude: The brain does not consider these logical equivalences as easy operations.

This cognitive conflict can be productively viewed in the light of the Mental Models theory, which is a well-known paradigm within the psychology of reasoning. Human thinking, according to this theory, is done by building and manipulating models of hypothetical worlds. When met with a conjunction, such as $\neg(A \wedge B)$, people generally form one mental model of the situation in which both A and B are true. To process the disjunction, then undo this conjunction operation (i.e., perform $\sim(A \wedge B)$), then also one model has to be generated and vicariously entertained; one with A false B true, A true B false, S-A-S-B false. This explosion of possibilities results in a 'Gödelian' explosion of mental models to be handled by the cognitive system. Conversion of a single model into multiple different models induces considerable cognitive load and the associated longer processing time and higher failure rate.

Contrast that with a disjunction like $(A \vee B)$: here the mind will attempt to build several models at first (A and its not-B, B and its not-A, etc). But to accept this disjunction $\neg(A \vee B)$ or $(A \vee C)$, or understand this denial, the mind has to eliminate those multiple representations and have one single (simpler) formula which is that of both A and B are false. This process of simplicity reduction, from the complex many to the neural one, is computationally "cheaper" and easier. The fact that the mind is so much more facile in manipulating the negation of a disjunction than it is in doing so with the negation of a conjunction serves to demonstrate that, when it comes to processing De Morgan's laws, mental operations are anything but cognitive symmetrical. As Macbeth et al. expression, "The tuning of these expressions to create more natural sentences is something which never completes or closes by any criteria, but it broken open over and over again in humans' interaction

⁶ John Inah Ukam, "Hilbert & Goedel on Completeness and Incompleteness Theory: A Look at the Problem of Formalism in Communication," *CRUTECH Journal of Communication* 4, no. 3 (2023): 1-15.
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and judgment.”⁷ This sentence highlights that logical thinking in humans is not something static or mechanical, but makes reference to a fluid and active (and often costly) process of meaning making.

This empirical picture is at odds with the formalist account of logic alone, which as Richard Zach puts it in his exposition of Hilbert’s program often aimed at reductive assimilation to a “soulless system of arbitrary symbols” that were substituted for variables and manipulated by rules thought to be completely deterministic.⁸ Instead the psychological evidence shows us that human logic is a profoundly embodied affair, circumscribed and heavily shaped by our biological and cognitive make-up. The “mental dynamics” which De Morgan himself sensed, is thus exposed to be a subtle interplay between formal rules and the incapacity/power of human beings to process information. It states that the “dynamism” of negation is not just a characteristic of formal systems, but a shaping force in how we think: our cognitive lives are spanned between (and depend upon the distance between) “disequilibrium and mitigation.”

5. Synthesis: Negation as a Social and Ethical Boundary

The problems that arise in applying rigorous, logical formalism to the dynamism of human negation (both in natural language and cognitive processes) come together to emphasize the importance of what seems like a ‘human element’. This is not a deficiency of the structure, however; it is the ultimate decider and coordinator of logical binary opposition. The situation is entirely reminiscent of Gödel’s incompleteness theorems, which state that a formal system, no matter how powerful, can never prove its own consistency without relying on assumptions outside itself. Consequently, no social or legal order, based upon formal rules and explicit negations, can arrive at perfect coherence nor to complete legitimation by its own immanent means. It has to keep making reference to an outside frame: that of the commonsensical instinct shared by all human beings, moral judgment and ability to pass beyond the boundaries of a system in order compare purposes and effects. This view is echoed now more than ever in debates about digital governance and in the current emerging environment of Decentralized Autonomous Organisations (DAOs). Underlying many DAOs is the fundamental belief, including in the so-called “code is law” mantra, that there are smart contracts and algorithmic rules that can drive collective action with minimal need for human discretion or interpretation. But such rigid code discipline frequently suffers a deep “De Morgan bind” when faced with the unexpected of new events or systemic distress. When a convoluted smart contract, basically a complicated web of ANDs and ORs is “negated” by an unforeseen exploit, market crash or new ethical conundrum - lack of inbuilt “adaptive governance” means the purely formal system often has no recourse to effectively deal with the aftermath.⁹ Even though the classical negation of a should-condition - in other words, the formal explication of an absence - within a system may technically be correct (from a logical point of view), it can and often does fall short with regard to what is meant or intimated by this act or its moral significance.

Indeed, as meticulously shown by Andrea Cesaretti in his beautiful study of the DAOs, a purely formal system is always caught between two horns: it can be “consistent” (that is, consistent with itself) only if its rules, pre-defined once for all and set down as would do Hamurabi’s dictums on clay tablets, take precedence over allowing justice to happen – or adapt up to some unknown extent with novel situations. On the other hand, it can aim for “completeness” by adding facilities that rely on human judgment and interpretation, but then you’re back in the world of humans.¹⁰ This tense relation is revealing of the fact that the formal architecture of negation, taken as such, and although certainly providing a structure for bounds and exclusions, is incomplete without its interpretive dimension—without an understanding human. The “negation of the collective intention” in these scenarios is not mere cognitive operation but a (highly complex) social act that requires an act of deliberation, consent by consensus and use of a value so far beyond algorithmic logic.

From this position, however, the supposed “incompleteness” or “resistance” of De Morgan’s laws when they are put into practice would not be a deficiency to overcome as much as an indication of a necessary restraint that any safeguarding of human freedom and dignity ought to have. Even if our social and juridical “negations”

⁷ Guillermo Macbeth, Eugenia Razumiejczyk, María del Carmen Crivello, Christian Bolzán, and Mauro Campitelli, “Mental Models for the Negation of Conjunctions and Disjunctions,” *Revista Argentina de Ciencias del Comportamiento* 6, no. 3 (2014): 17-25.

⁸ Richard Zach, “Hilbert’s Program,” in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta (Winter 2003), <https://plato.stanford.edu/entries/hilbert-program/>.

⁹ Charles E. A. Lincoln IV, “Axiomatic Shifting Paradigms: Wittgenstein’s Language-Games, Gödel’s Incompleteness Theorem, Language, Law, and the Limits of Formalism,” *University of Arkansas at Little Rock Law Review* 47, no. 2 (2025): 1-30.

¹⁰ Andrea Cesaretti, “Beyond the Code: Gödel’s Incompleteness and the Limits of Formal Governance in DAOs,” in *DAO Governance in Theory and Practice: Metrics and Frameworks* (Springer, 2025), 105-120. Available online at: <https://jazindia.com>

were strictly algorithmic, without human input, the construction of a “totalizing” system or network that computes meaning as such but provides no niche for emergent understanding or moral evolution would still be at stake. In acknowledging and accommodating the “Gödelian” fact of our institutions—that no formal system can be perfectly self-contained or self-justifying—we come to see that the “negation of collective intent” is essentially a construction (p. 190). “[Hammering the system open is] a contest to constantly spike the system open, with each generation finding more supple and creative ways to interpret and refuse it,” soches again. The great secret of wisdom must be learning when to use the beautiful, formal symmetries of De Morgan’s laws and when to listen to the “ghost” that directs us toward human context, intent, and the irreducible complexities of being alive. This nuanced dance keeps our systems in check while still being responsive, ethical, and ultimately human.

6. The Logic of Relations: De Morgan’s Extension of the Universe of Discourse

To truly understand the dynamic nature of negations in the De Morgan language, it is not satisfactory to stay on the level where propositions are formed; instead one has to consider his groundbreaking work on “the logic of relations.” De Morgan understood that the classic Aristotelian syllogism was seriously flawed insofar as it itself limited to monadic predicates—properties which apply to a single object (e.g., “Socrates is mortal”). He claimed that human thought is, for the most part, relational (e.g., “A is the father of B”) and therefore that negating such relations introduce additional complexities to logical dualism.¹¹

In his later memoirs published by the Cambridge Philosophical Society, De Morgan showed how negative English language (relations in fact) interact with relation composition. This “relational dynamics” generalizes his law-like principles to a many-dimensional space. For example, the negation of a complex relation—a disjunction (or) conjunctured with a conjunction (and), such as “person who is both colleague and friend”—calls for duality in something like its full glory as already in our modern-day relational databases and semantic networks. De Morgan’s realisation was that the “Universe of Discourse” does not (just) make up its own set of objects, but rather a network of relations, which can mean “better or worse cut” in practice, and that negation is the pivotal tool to trim and define the shapes around this network.

This view of relation further muddies the “isomorphism” of language and logic. Negations of relations tend to have “presuppositional” force in natural language. To say, “John is not Mary’s father” the denial here is not for its logical value only, but involves a category or relation in which assertion is possible. This “presuppositional dynamism” provides a further example of that “Gödelian” restriction in which the syntax of logic depends on an external framework of social and biological relations to be properly grasped. If we weave the logic of relations deeply into our grasp of De Morgan’s laws, we begin to recognize that this dynamism of the negative is more than merely a “passing over” from one form to another “side by side,” as it were; instead, negation works its way through our world not only in these two directions but also downwards – and it is this vertical play that opens up space between us.

7. Conclusion

The intellectual distance from Augustus De Morgan’s discovery that formal “inference”—logical deduction—could be understood as a form of calculus to our present-day grasp of the dynamism at work in negation is one of those vast and slow transformations in human idea-space. Finally, we have transcended mere pointwise algebraic interpretation of logical duality to understand its complex cross-domain behavior in negating conjunction and disjunction. That original hope for a closed, perfectly coherent logical universe, as appealing an ideal as it was, has given way to a certain kind of realism, the realization that the intricacies of human language and human cognition (not to mention human interaction) are inherently too complex ever to be fully formalized.

This paper has been a claim that De Morgan’s laws are not just something interesting to be known or attempted, but that their extremely critical rôle is as a lens for focusing an understanding of very awkward theoretical things. We have examined how the “linguistic break” undermines the simple applicability of these laws in natural language, where pragmatic and contextual conditions can leak through as they take precedence over strict truth-functionality. Moreover, the “cognitive cost” in processing negated conjunctions seen from the perspective of mental models theory makes it clear that logical operations are deeply embedded within human cognitive architecture, exerting a measurable friction on mental “exchange” necessary for De Morgan’s

¹¹ Augustus De Morgan, “On the Syllogism, No. IV, and on the Logic of Relations,” *Transactions of the Cambridge Philosophical Society* 10 (1864): 331-358.
Available online at: <https://jazindia.com>

equivalences. Last, the "social and ethical boundary" is a reminder that in trying to instantiate systems of pure reason -- such as digital governance -- ultimately we run up against Gödelian binds (what De Morgan calls mental assent) which necessarily require supplementation by human intuition, moral judgment and adaptive reading simply because they're incomplete.

Ultimately, the "Gödelian fracture" that we have discerned within De Morgan's program is not an omission but a universal feature of human enterprises. It represents the irreducible complexity of human action, context and creativity that can never be reduced to any finite set of formal rules. Whether developing systems of artificial intelligence that can work with natural language, laws and social contracts or robust digital communities; myopic focus on the illusion of perfect formal closure will not be enough. Instead, we should aim to construct systems that are not only coherent, but also highly "open", which mean able to entertain the "Gödelian ghost" -- that is the informalisable creative and context-embedded spirit of humanity. For good measure, De Morgan's laws offer us a remarkably clean blueprint of duality, a guide for exploring the foreboding logical landscape. But it is the human agency that brings this architecture to life and infuses this form with meaning, through which one reads in performance as well as values. In the end, the "negation" of our rules, of our presumptions, and of our formal borders is not to an endpoint but to a continuous new beginning—a creative act upon which we as human beings must embark again and anew if we are not just to rest on our laurels of truth, justice and meaning in a changing world.

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